

NAG Toolbox for MATLAB

f07fd

1 Purpose

f07fd computes the Cholesky factorization of a real symmetric positive-definite matrix.

2 Syntax

```
[a, info] = f07fd(uplo, a, 'n', n)
```

3 Description

f07fd forms the Cholesky factorization of a real symmetric positive-definite matrix A either as $A = U^T U$ if **uplo** = 'U' or $A = LL^T$ if **uplo** = 'L', where U is an upper triangular matrix and L is lower triangular.

4 References

Demmel J W 1989 On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $U^T U$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as LL^T , where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric positive-definite matrix A .

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

5.2 Optional Input Parameters

1: **n** – **int32 scalar**

Default: The second dimension of the array **a**.

n , the order of the matrix A .

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda

5.4 Output Parameters

1: **a(lda,*)** – **double array**

The first dimension of the array **a** must be at least $\max(1, n)$

The second dimension of the array must be at least $\max(1, n)$

The upper or lower triangle of A contains the Cholesky factor U or L as specified by **uplo**.

2: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **a**, 4: **lda**, 5: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** = i , the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A . To factorize a symmetric matrix which is not positive-definite, call f07md instead.

7 Accuracy

If **uplo** = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon |U^T| |U|,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*. If **uplo** = 'L', a similar statement holds for the computed factor L . It follows that $|e_{ij}| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to f07fd may be followed by calls to the functions:

f07fe to solve $AX = B$;

f07fg to estimate the condition number of A ;

f07fj to compute the inverse of A .

The complex analogue of this function is f07fr.

9 Example

```
uplo = 'L';
a = [4.16, 0, 0, 0;
     -3.12, 5.03, 0, 0;
      0.56, -0.83, 0.76, 0;
     -0.1, 1.18, 0.34, 1.18];
[aOut, info] = f07fd(uplo, a)

aOut =
    2.0396         0         0         0
   -1.5297    1.6401         0         0
    0.2746   -0.2500    0.7887         0
   -0.0490    0.6737    0.6617    0.5347
info =
      0
```
